

The Universality of Seesaws

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I discuss how the ideas associated with Seesaws, first introduced in the context of neutrino masses, are generally useful for understanding the very disparate scales one encounters in particle physics. From this point of view, the energy scale characterizing the Universe's dark energy presents a real challenge. A natural Seesaw explanation for this scale ensues if one imagines tying the dark energy sector to the neutrino sector, but this idea requires bold new dynamics.

1. A PLETHORA OF SCALES

One of the most difficult problems to understand in particle physics is the disparity of the mass scales that characterize fundamental interactions. A prototypical example is provided by the large hierarchy between the physical scale associated with gravitational interactions (the Planck mass, $M_P = G_N^{-1/2} = 1.22 \times 10^{19}$ GeV) and that connected with the electroweak interactions (the Fermi scale, $v_F = (\sqrt{2}G_F)^{-1/2} \simeq 250$ GeV):

$$M_P/v_F \simeq 10^{17}. \quad (1)$$

Another challenging issue is provided by the fine structure in the mass spectrum of quarks and charged leptons:

$$m_q = \{5 \text{ MeV} - 175 \text{ GeV}\} \quad (2)$$

$$m_\ell = \{0.5 \text{ MeV} - 2 \text{ GeV}\}, \quad (3)$$

whose origin remains a mystery. A third example is provided by the very small mass scales associated with neutrinos, compared to that of quarks and charged leptons:

$$m_\nu = \{4 \times 10^{-3} \text{ eV} - 2 \text{ eV}\}. \quad (4)$$

In this last example, however, it is possible to understand the general magnitude of m_ν from the Seesaw mechanism [1], whose twenty-fifth anniversary we are celebrating in this Symposium.

As a result of the Seesaw mechanism, neutrino masses are small because they reflect the presence of a much larger physical scale in the theory. Thus, typically,

$$m_\nu \sim v_F^2/M_N; \text{ or } m_\nu \sim v_F^2/M_X, \quad (5)$$

with M_N, M_X associated with either a very heavy right-handed neutrino $M_N \sim 10^{11} - 10^{15}$ GeV or the GUT scale $M_X \sim 10^{15} - 10^{16}$ GeV.

When thinking about the issue of scales in particle physics, traditionally one takes the Planck mass M_P as input and one asks questions about the origin of the light scales. There is a real plethora of such light scales, some arising from experimental input while others being pure theoretical constructs. I display some of these scales in Table 1. As one can see from this Table, these scales range over 30 orders of magnitude! Interrelating these scales is a real challenge and requires making assumptions on physics beyond the Standard Model. What I will try to argue here is that the existence of Seesaws may provide a useful guiding principle to help sort out what is fundamental and what is derivable among this panoply of scales.

The only scale in Table 1 which has a theoretically pristine origin is Λ_{QCD} , since this scale is set by the strong QCD dynamics itself. Roughly speaking, one can define Λ_{QCD} as the scale where the QCD coupling constant becomes strong: $\alpha_s(\Lambda_{QCD}^2) = 1$. The relation of Λ_{QCD}

Table 1
A sample of particle physics scales

Scale	Physics	Value (GeV)
M_P	Gravity	1.2×10^{19}
M_X	GUTS	2×10^{16}
M_N	RH neutrino	$10^{11} - 10^{15}$
f_{PQ}	PQ breaking	$10^9 - 10^{12}$
μ_S	SUSY breaking	$10^5 - 10^{15}$
v_F	EW breaking	250
M_H	Higgs	< 180
Λ_{QCD}	QCD	0.3
m_q	quarks	0.005-175
m_ℓ	leptons	5×10^{-4} - 2
m_ν	neutrinos	$10^{-12} - 10^{-9}$

to M_P is logarithmic and the only question really is why $\alpha_s(M_P^2) \simeq 1/45$. Could this be a boundary condition coming from physics at the Planck scale? Furthermore, because QCD is a dynamical theory, there is a close correlation between the physical scale Λ_{QCD} and the masses of the physical hadronic states. Indeed, $M_{\text{hadrons}} \sim \Lambda_{QCD}$.¹

The situation is much different in the electroweak theory. First, it is unlikely that the Fermi scale v_F is a dynamical scale, just like Λ_{QCD} , since precision electroweak experiments favor a light Higgs and disfavor QCD-like Technicolor theories. [2] Second, although m_q and m_ℓ are proportional to v_F , the fact that the mass spectrum of quarks and charged leptons spans five orders of magnitude suggests that the Yukawa couplings which engender these masses arise from physics at scales much larger than v_F . Third, and this is the real problem, in the Standard Model the sensitivity of v_F to any high energy cut-off is quadratic, so that the hierarchy $M_P/v_F \sim 10^{17}$ is very hard to understand.

I do not believe this hierarchy problem is resolved by extra-dimensional theories, [3] where one assumes that the Planck scale in (d+4)-dimensions is the Fermi scale: $M_P^d \simeq v_F$. These theories involve introducing a compactification radius R , whose scale is set by demanding that in 4-dimensions the scale of gravity is M_P . This

requires that

$$M_P = M_P^d (M_P^d R)^{d/2} \simeq v_F (v_F R)^{d/2}. \quad (6)$$

Obviously, the hierarchy problem $M_P/v_F \sim 10^{17}$ is replaced in these theories by the question of why $v_F R \gg 1$.

2. SEESAWS AS DYNAMICAL SOLUTIONS

In my view, it is much more satisfactory to think of v_F as originating from a Seesaw. This obtains in supersymmetric (SUSY) theories spontaneously broken in a hidden sector which is coupled to matter by gravity mediated interactions. [4] First of all, as is well known, the presence of a supersymmetry modifies the dependence of v_F on a high-energy cut-off from quadratic to logarithmic. Second, if indeed SUSY is spontaneously broken at a scale μ_S in a hidden sector coupled to matter only gravitationally, the superpartner masses (and other SUSY breaking parameters) are naturally given by a Seesaw formula:

$$\tilde{m} \simeq \mu_S^2 / M_P. \quad (7)$$

However, because of the large top Yukawa coupling, in this scenario [5] the breaking of supersymmetry naturally engenders a concomitant breaking of the electroweak theory. What happens, essentially, is that positive mass squared parameters in the Higgs sector at the SUSY breaking scale μ_S evolve at scales of $O(v_F)$ to negative

¹Pions are an exception because of their Nambu-Goldstone nature, with $m_\pi^2 \sim m_q \Lambda_{QCD}$.

mass squared parameters. That is,

$$\mu^2(\mu_S^2) \rightarrow -\mu^2(v_F^2). \quad (8)$$

Since $\mu^2(\mu_S^2) \sim \tilde{m}^2$, it follows that v_F is also given by a Seesaw formula:

$$v_F \simeq \mu_S^2/M_P. \quad (9)$$

For neutrinos, via the Seesaw mechanism, one obtains an understanding of the small scale for neutrino masses from the presence of a large new scale (either M_N or M_X) and a known intermediate scale v_F . If, indeed, the origin of the Fermi scale is due to the above SUSY induced Seesaw, we have effectively tied this scale $v_F \simeq 250$ GeV to a much larger scale $\mu_S \sim 10^{11}$ GeV. In this Seesaw, in contrast to the neutrino case, we have used **known** low and high scales (v_F and M_P , respectively) to infer the existence of an intermediate scale μ_S .

From a fundamental point of view, there might appear to be no real advantage to having "explained" v_F through the introduction of a new intermediate scale μ_S , except to have shortened the gap between the Planck scale and the driving scale μ_S for the physics we observe. The hierarchy to explain now is not $M_P/v_F \sim 10^{17}$ but $M_P/\mu_S \sim 10^8$. However, in the literature, many attempts exist to use the scale μ_S also as the prototypical scale where family structure originates. [6] Typically, this is done by exploiting variants of the Froggatt-Nielsen [7] mechanism, with small Yukawa couplings being given by VEV ratios, $\Gamma \sim [\langle \sigma \rangle / \mu_S]^n$, with the VEV $\langle \sigma \rangle$ breaking some assumed family symmetry.

I will not pursue these matters further here, except to remark that one can systematically relate most of the scales that enter in Table 1 to physics at higher scales via some kind of Seesaw formulas.² Instead I want to spend some time discussing how these ideas of having Seesaw formulas tie low energy parameters to physics at higher scales runs

²As an example, for instance, the axion mass is given by the Seesaw formula $m_a \sim m_q^{1/2} \Lambda_{QCD}^{3/2} / f_{PQ}$ involving the ratio of the scale where the axial anomaly becomes relevant and the scale where $U(1)_{PQ}$ breaks down spontaneously. The presence of the quark mass in the formula for m_a reflects the influence of additional chiral symmetries in QCD in the massless quark limit.

into a significant challenge when one tries to address the issue of dark energy in the Universe.

3. DIALING SCALES THROUGH THE UNIVERSE

Einstein's equations describing the expansion of the Universe in a Robertson-Walker background provide a wonderful scale-meter. The Hubble parameter at different temperatures during the expansion provides the yardstick. Although now $H_o = (1.5 \pm 0.1)10^{-33}$ eV is a tiny scale, since the value of the Hubble parameter varies with temperature as the Universe cools it samples all scales, from the Planck mass downwards.

Einstein's equations, written in terms of the Robertson-Walker scale factor R

$$H^2 \equiv \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G_N \rho}{3} - \frac{k}{R^2} + \frac{\Lambda}{3} \quad (10)$$

$$\frac{\ddot{R}}{R} = \frac{\Lambda}{3} - \frac{4\pi G_N}{3}(\rho + 3p) \quad (11)$$

determine H and the Universe's acceleration once ρ , p , k and Λ are specified. In a flat Universe [$k = 0$], as predicted by inflation [8] and confirmed observationally by WMAP, [9] the Universe's expansion accelerates if the cosmological constant is such that $\Lambda > 4\pi G_N \rho_{\text{matter}}$. Alternatively, if $\Lambda = 0$, the expansion of the Universe accelerates if a dominant component of the Universe has negative pressure and $\rho + 3p < 0$. The observed acceleration is evidence for this dark energy.

It is convenient in what follows to set the parameter $\Lambda = 0$ adding, however, a dark energy contribution explicitly to the density. That is, $\rho \rightarrow \rho + \rho_{\text{dark energy}}$. It is easy to see then that the pure cosmological constant case corresponds to assuming the equation of state

$$\omega_{\text{dark energy}} = \frac{p_{\text{dark energy}}}{\rho_{\text{dark energy}}} = -1, \quad (12)$$

where $\rho_{\text{dark energy}} = \rho_{\text{vacuum}} = \text{constant}$. Writing

$$H^2 = \frac{8\pi G_N \rho}{3} + \frac{8\pi G_N \rho_{\text{dark energy}}}{3} \quad (13)$$

we know observationally that, at the present time, H_o^2 gets about 30% contribution from the first

term and 70% from the second. So there appear to be two Seesaws to explain:

$$H_o \simeq \frac{\rho_o^{1/2}}{M_P}; \text{ and } H_o \simeq \frac{\rho_{\text{dark energy}}^{1/2}}{M_P}. \quad (14)$$

The first Seesaw, which in fact is not a true Seesaw, is understood in terms of known, or at least speculated, physics. The other Seesaw, involving the dark energy density, is totally mysterious.

Let me try to address both points below. Because the energy density depends on the Universe's scale factor R as

$$\rho \sim R^{-3(1+\omega)}, \quad (15)$$

the contribution of $\rho_{\text{dark energy}}$ to H^2 at earlier times is negligible, so that

$$H^2 \simeq \frac{8\pi G_N \rho}{3}. \quad (16)$$

Because the Hubble parameter depends on temperature, $H = H(T)$, the above is really a dynamical equation, not a Seesaw. The total density ρ just fixes the rate of the Universe's expansion.

As the Universe expands different components dominate ρ , as they, in general, have different temperature dependences and different threshold factors. Schematically, one can write

$$\rho = \rho_{\text{radiation}} + \rho_{\text{matter}} + \rho_{\text{dark matter}}, \quad (17)$$

with

$$\rho_{\text{radiation}} = \left[\frac{\pi^2}{30}\right] g(T) T^4 \quad (18)$$

$$\rho_{\text{matter}} = \left[\frac{2\xi(3)}{\pi^2}\right] \{M_B \eta + \Sigma_i m_{\nu_i}\} T^3 \quad (19)$$

$$\rho_{\text{dark matter}} \simeq \left\{ f_{PQ} \Lambda_{QCD} + \frac{m^*}{T^* < \sigma v >} \right\} \frac{T^3}{M_P}. \quad (20)$$

In the above, the different components in ρ become effective when the Universe cools below the relevant decoupling temperature. For instance, baryons contribute after nucleosynthesis starts ($T_{\text{decoupling}} \sim \text{MeV}$); neutrinos turn on below $T \simeq m_{\nu_i}$; axionic dark matter, if it exists, turns on below the QCD phase transition; and neutralino dark matter, if it exists, turns on below the electroweak phase transition. At any rate,

at present [$T_o \simeq 3^\circ K$; $g(T_o) = 2$] the contribution of $\rho_{\text{radiation}}$ is negligible, while the particle physics parameters $M_B, \eta, m_{\nu_i}, f_{PQ}$, etc insure that ρ_{matter} and $\rho_{\text{dark matter}}$ contribute, respectively, approximately 2% and 28% to H_o^2 .

The situation is quite different with the second Seesaw. Here, for example, if the dark energy is due to the presence of a cosmological constant, so that $\rho_{\text{dark energy}} = \rho_{\text{vacuum}} = E_o^4$, one has a real Seesaw:

$$H_o \simeq \frac{E_o^2}{M_P}, \quad (21)$$

which fixes $E_o \simeq 2 \times 10^{-3} \text{ eV}$. What physics is associated with this small energy scale? All vacuum energies we know in particle physics are enormously bigger. For instance, $E_o^{QCD} \sim \Lambda_{QCD} \sim 1 \text{ GeV}$.

Matters are not substantially different if $\rho_{\text{dark energy}}$ has a more dynamical origin. Although now the dark energy is temperature dependent

$$\rho_{\text{dark energy}} = \rho_{\text{dark energy}}^o [T/T_o]^{3(1+\omega)}, \quad (22)$$

the parameters characterizing the dynamical theory are very difficult to understand. Let me illustrate this point by considering quintessence [10] as a model for the dark energy of the Universe.

In this case one associates the dark energy with a new scalar field ϕ which has negative pressure. In the present epoch we know that $\rho_{\text{dark energy}} \simeq 0.7[3H_o^2/8\pi G_N]$ and that the dark energy equation of state gives a range for $\omega_{\text{dark energy}}$ between -0.7 and -1.2. [11]. Thus, if quintessence is the source for the dark energy, we must have that

$$\rho_{\text{quint}} = \frac{\dot{\phi}^2}{2} + V(\phi) \simeq 0.7 \left[\frac{3H_o^2}{8\pi G_N} \right] \quad (23)$$

$$p_{\text{quint}} = \frac{\dot{\phi}^2}{2} - V(\phi) \simeq 0.7 \omega \left[\frac{3H_o^2}{8\pi G_N} \right]. \quad (24)$$

The field ϕ is dynamical and to realize the above equations the magnitude of the field ϕ must be large, of order of the Planck mass itself: $\phi \sim M_P$. With such large fields it is impossible to obtain the above results for ρ and p unless the field ϕ has nearly zero mass:

$$m_\phi \sim \frac{E_o^2}{\phi} \sim H_o \simeq 10^{-33} \text{ eV}. \quad (25)$$

The above Seesaw formula, however, is unprotected from getting big mass shifts, unless the quintessence field essentially decouples from matter. [12]

4. NEUTRINOS TO THE RESCUE?

In a sense, the quintessence interpretation of $\rho_{\text{dark energy}}$ results in a very unpalatable Seesaw:

$$m_\phi \simeq \frac{E_o^2}{M_P}, \quad (26)$$

where a difficult to understand scale $E_o \sim 2 \times 10^{-3}$ eV produces, from a particle physics point of view, an even more difficult to understand scale $m_\phi \sim H_o \simeq 10^{-33}$ eV. It would be much more satisfactory if one could understand the dark energy density as arising dynamically from a known particle physics scale.

A very interesting suggestion along these lines has been put forward recently by Fardon, Nelson and Weiner.[13] Its starting premise is that one should be able to explain in a natural fashion why, in the present epoch, the energy density associated with dark energy and matter should be approximately the same: $\rho_{\text{dark energy}} \simeq \rho_{\text{matter}}$. Because these densities, presumably, have different temperature dependences, their near equality now itself is a mystery. As many people have noted, this coincidence is resolved dynamically if the dark energy density tracks (some component) of the matter density. [14] What Fardon, Nelson and Weiner suggest is that $\rho_{\text{dark energy}}$ tracks the energy density in neutrinos, ρ_ν . This avoids some of the issues that would arise if $\rho_{\text{dark energy}}$ were really to track some better known component of ρ_{matter} , like ρ_B . Further it perhaps allows one to understand the scale E_o associated with $\rho_{\text{dark energy}}$ in terms of the scale of neutrino masses, which are of a similar magnitude.

The idea put forth by Fardon, Nelson and Weiner [FNW] [13] is quite radical. By imagining that the neutrinos and the dark energy are coupled together, the energy density associated with the dark energy depends on the neutrino masses. In turn, these masses are not fixed but are variable, with their magnitude being a function of the neutrino density, $m_\nu = m_\nu(n_\nu)$. Assuming

for simplicity just one flavor of neutrino, in the FNW picture the energy density in the dark sector (neutrinos plus dark energy) is given by:

$$\rho_{\text{dark}} = m_\nu n_\nu + \rho_{\text{dark energy}}(m_\nu). \quad (27)$$

This energy density will stabilize when

$$n_\nu + \rho'_{\text{dark energy}}(m_\nu) = 0. \quad (28)$$

The above equations have an immediate implication for the equation of state in the dark sector. Since

$$\omega + 1 = -\frac{\partial \ln \rho_{\text{dark}}}{3 \partial \ln R}, \quad (29)$$

it follows that

$$\omega + 1 = -\left[\frac{R}{3\rho_{\text{dark}}}\right]\left\{m_\nu \frac{\partial n_\nu}{\partial R} + n_\nu \frac{\partial m_\nu}{\partial R} + \rho'_{\text{dark energy}} \frac{\partial m_\nu}{\partial R}\right\}, \quad (30)$$

whence

$$\omega + 1 = \frac{m_\nu n_\nu}{\rho_{\text{dark}}} = \frac{m_\nu n_\nu}{m_\nu n_\nu + \rho_{\text{dark energy}}}. \quad (31)$$

We see from this equation that if $\omega \simeq -1$, the neutrino contribution to ρ_{dark} is a small fraction of $\rho_{\text{dark energy}}$. Further, since we expect that $\rho_{\text{dark energy}} \sim R^{-3(1+\omega)}$, if ω does not change significantly with R , it follows that the neutrino mass is nearly **inversely proportional** to the neutrino density:

$$m_\nu \sim n_\nu^\omega. \quad (32)$$

I will not discuss the FNW scenario much further here, but will make just a few remarks:

1. If ω is near its central value, $\omega = -0.8$, then the dark sector equation of state predicts that in the present epoch $m_\nu^{\text{cosmo}} \simeq 5$ eV. However, since $m_\nu \sim n_\nu^\omega$, if there is neutrino clustering in our galaxy, the observed neutrino mass on earth could be much smaller: $m_\nu^{\text{obs}} \simeq 5[n_\nu^{\text{local}}/n_\nu^{\text{cosmo}}]^\omega$ eV.
2. The variability of neutrino masses with their density requires re-examining many of the astrophysical and cosmological constraints on neutrinos coming from big bang

nucleosynthesis, supernovas, leptogenesis, etc. Remarkably, the FNW scenario seems to survive rather unscathed by these constraints. [13] [15]

3. Although the detailed dynamics of the dark sector is unclear at this stage, it is most likely that the coupling between dark energy and neutrinos comes through the $SU(2) \times U(1)$ singlet M_N . So, the neutrino masses feel the field responsible for the dark sector dynamics, $\phi_{\text{dark energy}}$, via the Seesaw: $m_\nu \sim v_F^2/M_N(\phi_{\text{dark energy}})$.
4. This scenario naturally "explains" why the energy scale associated with the dark energy, E_o , is of the same order as that of neutrino masses, since these components of the Universe track each other. Indeed, E_o is set by the same Seesaw which gives neutrino masses: $E_o \sim m_\nu \sim v_F^2/M_N(\phi_{\text{dark energy}})$.

5. CONCLUDING REMARKS

I hope that the discussion presented above has helped to show that the ideas associated with Seesaws are useful when one wants to reach some understanding of the very disparate scales one encounters in particle physics. In fact, most if not all of the scales entering particle physics can be associated, in one form or another, to a Seesaw. So, in a sense, Seesaws provide a universal framework to think about disparate scales.

From this point of view, the dark energy scale $E_o \sim 2 \times 10^{-3}$ eV presents a real challenge. Straightforward particle physics models of dark energy, like quintessence, [10] involve even less understandable mass scales, which cannot be protected unless quintessence decouples from everything else.[12] The FNW scenario just discussed [13] allows a natural Seesaw explanation for E_o . However, the idea of tying the dark energy sector to the neutrino sector is very speculative and requires imagining bold new dynamics.

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